Exercice 1 — Partie A — $X \sim \mathcal{P}(500\tau)$

1 - $\tau = 0{,}01$ — $X \sim \mathcal{P}(5)$ (dans la table)

a - $p(X \leq 1) = 0{,}007 + 0{,}034 = \boxed{0{,}041}$

b - $p(X > n_0) < 0{,}05 \iff 1 - p(X \leq n_0) < 0{,}05 \iff p(X \leq n_0) > 0{,}95$

on ajoute, dans la colonne $\lambda = 5$, les $p(X = k)$ jusqu'à dépasser 0,95

k = 8 $\quad 0{,}007 + 0{,}034 + 0{,}084 + 0{,}140 + 0{,}176 + 0{,}176 + 0{,}146 + 0{,}104 + 0{,}065 = 0{,}932$

k = 9 $\quad$ " " " $+ 0{,}036 = 0{,}968$

donc $\boxed{n_0 = 9}$

2 - $\tau = 0{,}2$ — $X \sim \mathcal{P}(100) \sim \mathcal{N}(100, 10)$

a - $p(X > 120) = p\left(\frac{X-100}{10} > \frac{120-100}{10}\right) = p\left(\frac{X-100}{10} > 2\right) = 1 - \Pi(2) = 1 - 0{,}9772 = \boxed{0{,}0228}$

b - $p(100 - a \leq X \leq 100 + a) = 0{,}99 \iff p\left(\frac{100-a-100}{10} \leq \frac{X-100}{10} \leq \frac{100+a-100}{10}\right) = 0{,}99$

$\iff p\left(-\frac{a}{10} \leq \frac{X-100}{10} \leq \frac{a}{10}\right) = 0{,}99 \iff 2\Pi\left(\frac{a}{10}\right) - 1 = 0{,}99 \iff \Pi\left(\frac{a}{10}\right) = \frac{1{,}99}{2} = 0{,}995$

$\iff \frac{a}{10} \simeq 2{,}58 \qquad \boxed{a = 25{,}8}$

Partie B — 1 - on répète $n = 30$ fois de façon indépendante une expérience aléatoire à 2 issues, le succès (dysfonctionnement) de probabilité $p = 0{,}01$ et l'échec de probabilité $q = 0{,}99$

a - donc $Y \sim \boxed{\mathcal{B}(30\,;0{,}01)}$

b - $p(Y \leq 2) = \binom{30}{0} 0{,}01^0\, 0{,}99^{30} + \binom{30}{1} 0{,}01^1\, 0{,}99^{29} + \binom{30}{2} 0{,}01^2\, 0{,}99^{28} \simeq \boxed{0{,}997}$ à 10^{-3} près

2 - a - $Z \sim \boxed{\mathcal{B}(365\,;0{,}01)}$ $\quad n = 365 \quad p = 0{,}01$

b - $E(Z) = np = \boxed{3{,}65}$ $\qquad \sigma(Z) = \sqrt{npq} \simeq \boxed{1{,}901}$

Partie C — 1 - $p(T \leq t) = \int_0^t 500 e^{-500x}\,dx$

a - $p(T \leq t) = \left[\frac{500 e^{-500x}}{-500}\right]_0^t = \left[-e^{-500x}\right]_0^t = \boxed{1 - e^{-500t}}$

b - $1 - e^{-500t} = 0{,}95 \iff e^{-500t} = 0{,}05 \iff -500t = \ln 0{,}05$

$\iff \boxed{t = -\frac{\ln 0{,}05}{500} \simeq 0{,}006}$ à 10^{-3} près.

2 - a - $I(t) = \int_0^t 500x\, e^{-500x}\,dx$.

$u(x) = 500x \qquad u'(x) = 500$

$v'(x) = e^{-500x} \qquad v(x) = \frac{e^{-500x}}{-500}$

$I(t) = \left[-x e^{-500x}\right]_0^t + \int_0^t e^{-500x}\,dx = \left[-x e^{-500x}\right]_0^t + \left[\frac{e^{-500x}}{-500}\right]_0^t$

$\boxed{I(t) = -t e^{-500t} - \frac{e^{-500t}}{500} + \frac{1}{500}}$

b - $\lim_{t \to +\infty} I(t) = \boxed{m = \frac{1}{500}}$ (croissances comparées)

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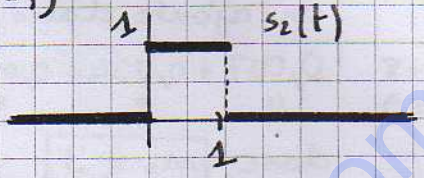
Exercice 2

Partie A $s_1(t) + \int_0^t s_1(u)\,du = \mathcal{U}(t)$

1 – $S_1(p) + \dfrac{S_1(p)}{p} = \dfrac{1}{p} \iff S_1(p)\left(1 + \dfrac{1}{p}\right) = \dfrac{1}{p} \iff \boxed{S_1(p) = \dfrac{1}{p+1}}$

2 – $\boxed{s_1(t) = e^{-t}\,\mathcal{U}(t)}$ car $\mathcal{L}(e^{-at}\mathcal{U}(t)) = \dfrac{1}{p+a}$ ici $a = 1$.

Partie B $s_2(t) + \int_0^t s_2(u)\,du = \mathcal{U}(t) - \mathcal{U}(t-1)$



1 – $t < 0$ $\mathcal{U}(t) = \mathcal{U}(t-1) = 0$ $s_2(t) = 0$
$0 \le t < 1$ $\mathcal{U}(t) = 1$ $\mathcal{U}(t-1) = 0$ $s_2(t) = 1$
$1 \le t$ $\mathcal{U}(t) = \mathcal{U}(t-1) = 1$ $s_2(t) = 0$

2 – $S_2(p) + \dfrac{S_2(p)}{p} = \dfrac{1}{p} - \dfrac{1}{p}e^{-p}$ car $\mathcal{L}(f(t-\tau)\mathcal{U}(t-\tau)) = F(p)e^{-\tau p}$
ici $\tau = 1$ $f(t) = \mathcal{U}(t)$

$\boxed{S_2(p) = \dfrac{1}{p+1} - \dfrac{1}{p+1}e^{-p}}$

3 – a – $\boxed{s_2(t) = e^{-t}\mathcal{U}(t) - e^{-(t-1)}\mathcal{U}(t-1)}$ car $\mathcal{L}(f(t-\tau)\mathcal{U}(t-\tau)) = F(p)e^{-\tau p}$
avec $\tau = 1$ $F(p) = \dfrac{1}{p+1}$ $f(t) = e^{-t}\mathcal{U}(t)$

b – $t < 0$ $s_2(t) = \boxed{0}$
$0 \le t < 1$ $s_2(t) = \boxed{e^{-t}}$
$1 \le t$ $s_2(t) = e^{-t} - e^{-(t-1)} = e^{-t}e^{-t+1} = e^{-t}(1-e) = \boxed{-e^{-t}(e-1)}$

4 – sur $]1, +\infty[$ $s'_2(t) = e^{-t}(e-1) > 0$ donc $\boxed{s_2 \text{ croissante}}$ sur $[1, +\infty[$

5 – $s_2(1^+) - s_2(1^-) = -e^{-1}(e-1) - e^{-1} = \boxed{-1}$ (valeur de la discontinuité)

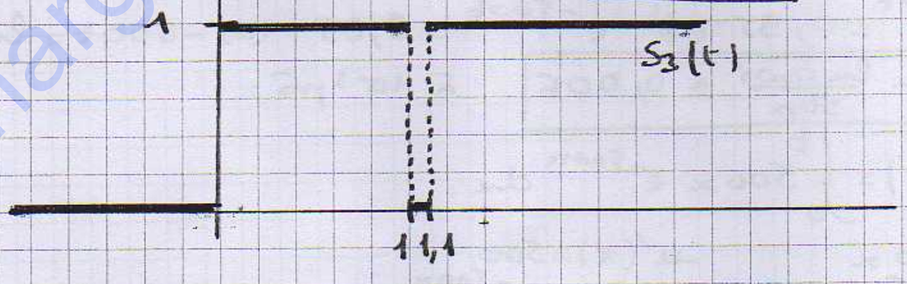
6 – a –

t	1	1,1	1,5	2	2,5
s_2	−0,63	−0,57	−0,38	−0,23	−0,14

Partie C $s_3(t) + \int_0^t s_3(u)\,du = \underbrace{\mathcal{U}(t) - \mathcal{U}(t-1) + \mathcal{U}(t-1{,}1)}_{e_3(t)}$

a – $t < 0$ $\mathcal{U}(t) = \mathcal{U}(t-1) = \mathcal{U}(t-1{,}1)$ $\boxed{s_3(t) = 0 = s_2(t)}$
$0 \le t < 1$ $\mathcal{U}(t) = 1$ $\mathcal{U}(t-1) = \mathcal{U}(t-1{,}1) = 0$ $\boxed{s_3(t) = 1 = s_2(t)}$
$1 \le t < 1{,}1$ $\mathcal{U}(t) = \mathcal{U}(t-1) = 1$ $\mathcal{U}(t-1{,}1) = 0$ $\boxed{s_3(t) = 0 = s_2(t)}$

b – $1{,}1 \le t$ $\mathcal{U}(t) = \mathcal{U}(t-1) = \mathcal{U}(t-1{,}1) = 1$ $\boxed{s_3(t) = 1}$



2 – $s_3(t) = \begin{cases} 0 & t \le 0 \\ e^{-t} & 0 \le t < 1 \\ -e^{-t}(e-1) & 1 \le t < 1{,}1 \\ e^{-t}(1 - e + e^{1,1}) & 1{,}1 \le t \end{cases}$

sur $[1{,}1 ; +\infty[$ $s'_3(t) = -e^{-t}(1 - e + e^{1,1}) < 0$

$\boxed{s_3 \text{ décroissante}}$ sur $[1{,}1 ; +\infty[$

3 – $s_3(1{,}1^+) - s_3(1{,}1^-) = \boxed{1}$
valeur de la discontinuité

t	1,1	1,5	2	2,5
s_3	0,43	0,29	0,17	0,11

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Document réponse, à rendre avec la copie (exercice 2)

Figure 1 : représentation de la fonction s_1

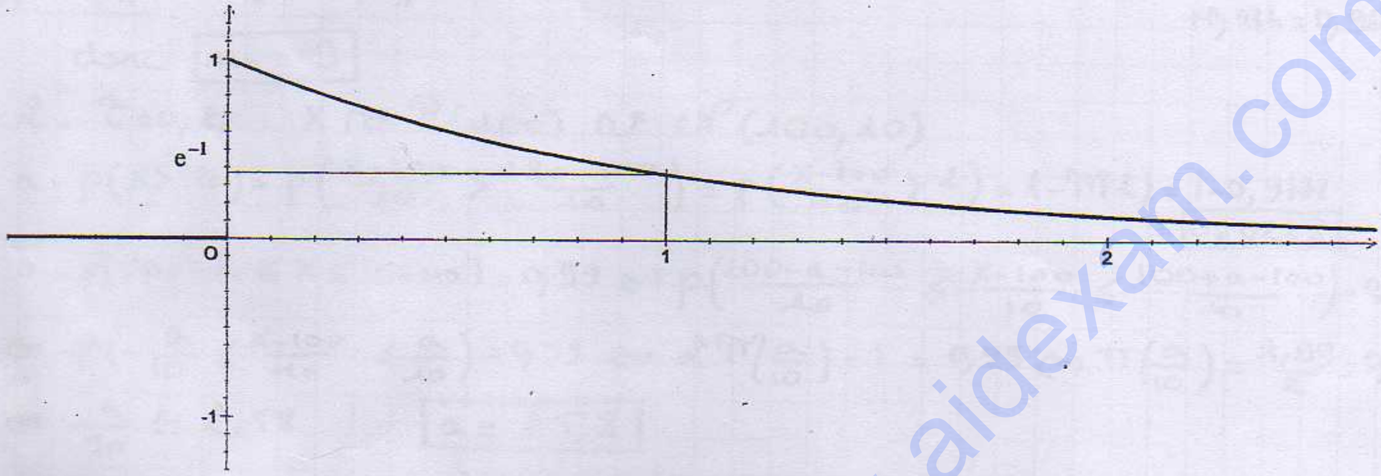
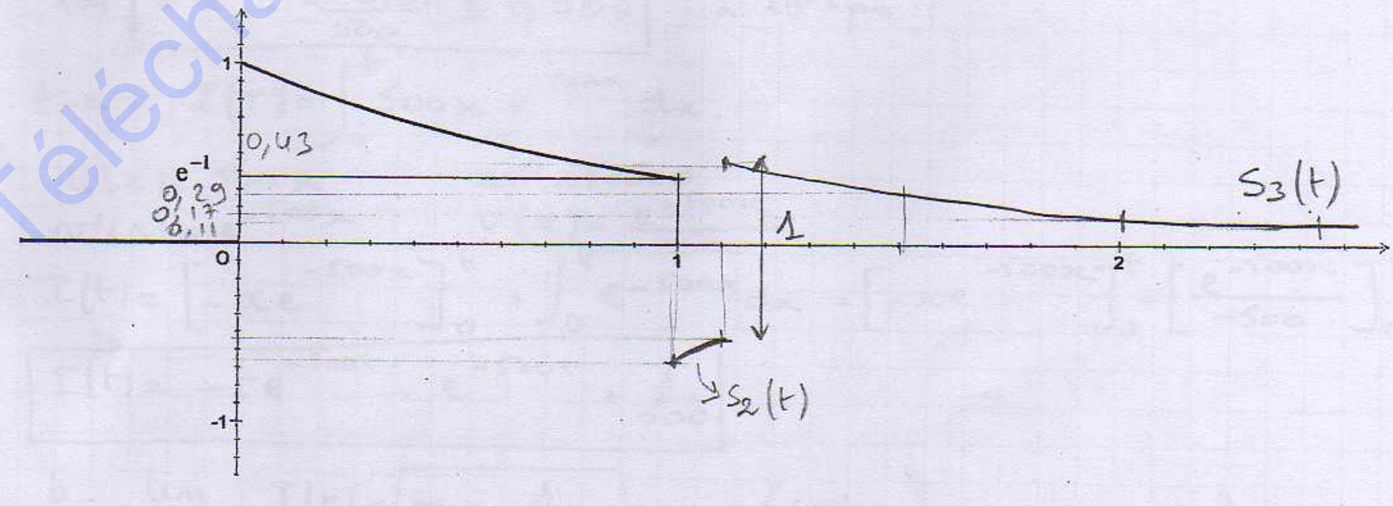


Figure 2 : représentation de la fonction s_2 à compléter

Figure 3 : représentation de la fonction s_3 à compléter



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